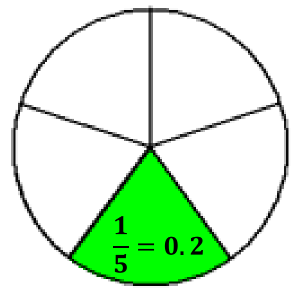
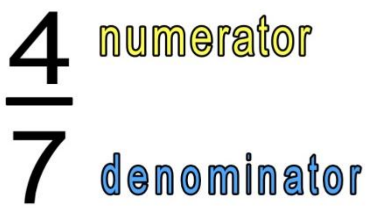
Section 3.1 – Fractions to Decimals

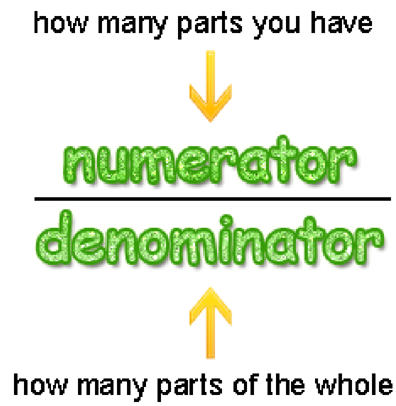
A fraction is a part of a whole. For example, $\frac{1}{5}$ is a fraction; it means 1 out of 5 possible pieces.



Fractions also illustrate division. For example, $\frac{1}{5}$ also means $1 \div 5$ which equals 0.2.

Every fraction has a **numerator** (top number) and a **denominator** (bottom number).





All fractions can be written as either ***terminating*** or ***repeating*** decimals; that is, when we divide the numerator by the denominator the digits in the answer will either terminate or repeat.

Terminating Decimals

The fractions below,

$\frac{1}{2}$ = **0.5**, $\frac{1}{5}$ = **0.2** and $\frac{3}{4}$ = **0.75,**

terminate since they have a finite number of digits after the decimal point (they stop).

Repeating Decimals

The following fractions,

$\frac{1}{9}$ = **0.11111....**, $\frac{2}{11}$ = **0.181818...** and $\frac{3}{7}$ = **0.428571428...**

are **repeating decimals** since a digit or block of digits after the decimal point repeats without end.

A REPEATING DECIMAL
is a decimal that repeats a digit (or digits) over and over again.

We can write a bar above the repeating digits to indicate repetition.

For example,

0.11111.... = $\mathbf{0.\overline{1}}$, 0.181818... = $\mathbf{0.\overline{18}}$ and 0.428571428... = $\mathbf{0.\overline{428571}}$

Example: Using a calculator, change the following fractions into decimals and tell if it is repeating or terminating.

a) $\frac{3}{5}$

b) $\frac{7}{8}$

c) $\frac{1}{3}$

d) $\frac{13}{16}$

e) $\frac{17}{21}$

f) $\frac{22}{37}$

<u>Example 2</u>: Patterns sometimes occur when we write fractions in decimal form. Using a calculator, change the following fractions into decimals and tell if it is repeating or terminating. What do you notice?

a) $\frac{5}{9}$

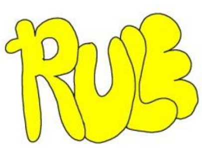
b) $\frac{124}{999}$

c) $\frac{7}{99}$

d) $\frac{67}{99}$

e) $\frac{7}{999}$

f) $\frac{1234}{9999}$



What rule can we write for changing fractions into decimals that have a denominator of 9, 99, 999 etc?

<u>Example 3:</u> Given the pattern $\frac{1}{11} = 0.\overline{09}$, $\frac{2}{11} = 0.\overline{18}$, $\frac{3}{11} = 0.\overline{27}$

a) Determine the decimals for $\frac{5}{11}$ and $\frac{9}{11}$

b) What fraction will have 0.636363… as a decimal?

<u>Fractions with Denominators of 10, 100, 1000</u>

A fraction with a denominator of 10, 100 or 1000 can be easily converted into a decimal.

The number of zeros indicates the number of places the decimal needs to shift to the left in the numerator.

For example, consider $\frac{3}{10}$. In this case the numerator is 3 or 3.0.
Since there is one zero in the denominator, move the decimal one place to the left.

$\frac{3}{10}$ **has one zero in the denominator** This means,

Move one decimal place

$3.0 \quad = \quad 0.3$

Let's consider $\frac{57}{100}$

$\frac{57}{100}$ **has two zeros in the denominator** This means,

Move two decimal places

$57.0 \quad = 0.57$

<u>Example 1:</u> Write the decimal equivalent for each fraction.

a) $\frac{5}{10}$ b) $\frac{79}{10}$ c) $\frac{90}{100}$ d) $\frac{1735}{1000}$

e) $\frac{56}{100}$ e) $\frac{326}{1000}$ f) $\frac{235}{10}$ g) $\frac{752}{100}$

If the fraction does not have a denominator of 10, 100 or 1000, we try to change the denominator to 10, 100 or 1000, and then write as a decimal.

Let's consider $\frac{4}{5}$. Can we easily change the denominator to 10, 100 or 1000?

$\frac{4}{5}$ can be easily changed to $\frac{8}{10}$ by multiplying the numerator and denominator by 2.

Therefore, $\frac{8}{10} = 0.8$.

<u>Example 2:</u> If possible, write each of the following fractions with a denominator of 10, 100 or 1000 and then write the decimal equivalent.

a) $\frac{3}{25}$ b) $\frac{7}{20}$ c) $\frac{19}{50}$

d) $\frac{22}{200}$ e) $\frac{324}{500}$ f) $\frac{7}{125}$

Reducing Fractions

Sometimes we may be asked to reduce, that is, make the fraction smaller.

To do this, we find the biggest number that divides evenly into the numerator and denominator.

For example, consider $\frac{100}{200}$.

The biggest number that divides evenly into 100 and 200 is 100.

Therefore, $\frac{100}{200} \quad \frac{\div 100}{\div 100} = \frac{1}{2}$

Example: Write each fraction in simplest form.

a) $\frac{10}{25}$ b) $\frac{16}{20}$ c) $\frac{22}{50}$

d) $\frac{24}{30}$ e) $\frac{12}{60}$ f) $\frac{60}{90}$

Writing Decimals as Fractions

To write a decimal as a fraction, we count the number of decimal places to the right of the zero…that's how many zeros get placed in the denominator. We then reduce the fraction if possible.

Let's consider 0.55,

0.55

two numbers after the decimal

This means, $\frac{55}{100}$ **Two zeros in the denominator**

$$\frac{55 \div 5}{100 \div 5} = \frac{11}{20}$$

This can be reduced!

Example: Write each decimal in fractional form. Reduce if possible.

a) 0.1

b) 0.05

c) 0.555

d) 0.35

e) $0.\overline{3}$

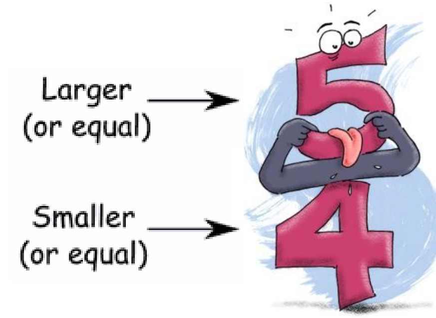
f) $0.\overline{98}$

g) $0.\overline{027}$

h) $0.\overline{09}$

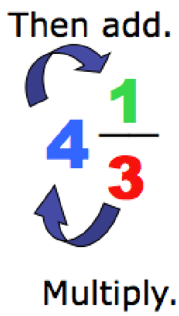
Mixed Numbers and Improper Fractions

An improper fraction exists when the numerator is larger than the denominator.



To write a mixed number as an improper fraction, we:

1. Multiply the denominator by the whole number and add the numerator.
2. Keep the denominator the same.



$4 \times 3 = 12 + 1 = 13$

Our answer is $\frac{13}{3}$

Example: Write each mixed number as a improper fraction:

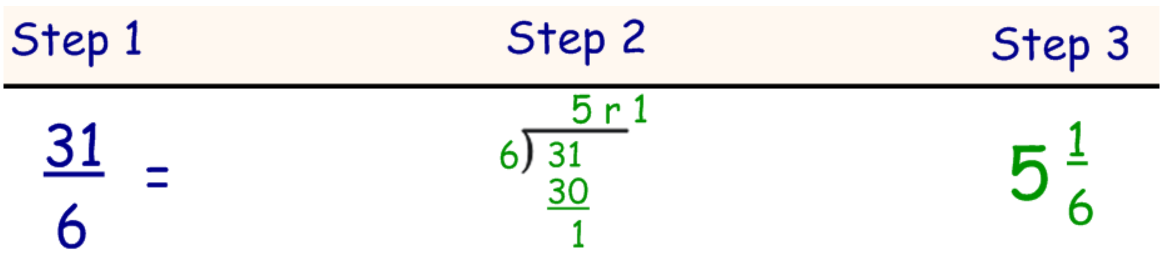
a) $3\frac{2}{5}$ b) $2\frac{5}{7}$ c) $3\frac{1}{3}$

d) $1\frac{3}{4}$ e) $3\frac{5}{6}$ f) $5\frac{1}{8}$

To write an improper fraction as a mixed number we do the reverse:

1. Find out how many times the denominator goes into the numerator. This becomes our whole number.

2. Whatever is "leftover" goes over the denominator as a fraction.



Example: Write each improper fraction as a mixed number.

a) $\frac{15}{7}$ b) $\frac{23}{5}$ c) $\frac{21}{2}$

d) $\frac{19}{6}$ e) $\frac{9}{2}$ f) $\frac{45}{8}$

Section 3.2 – Comparing and Ordering Fractions and Decimals

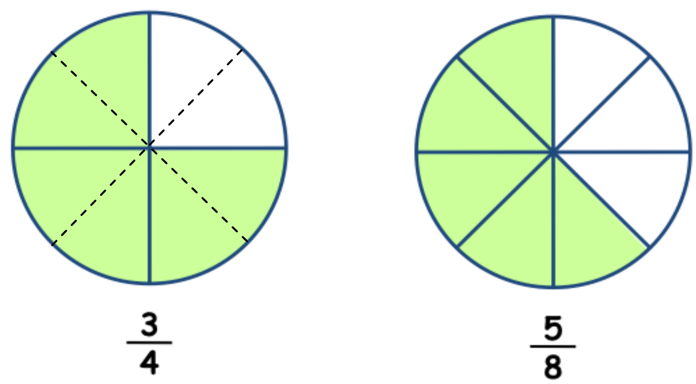
We will use several methods to compare and order fractions:

1. Model fractions and/or decimals using blocks, fraction pieces, pattern blocks, etc.
2. Cross Multiplying (Butterfly Method)
3. Changing to decimals and comparing using place value
4. Comparing using benchmarks such as 0, $\frac{1}{2}$ or 1
5. Comparing common numerators or denominators using equivalent fractions.

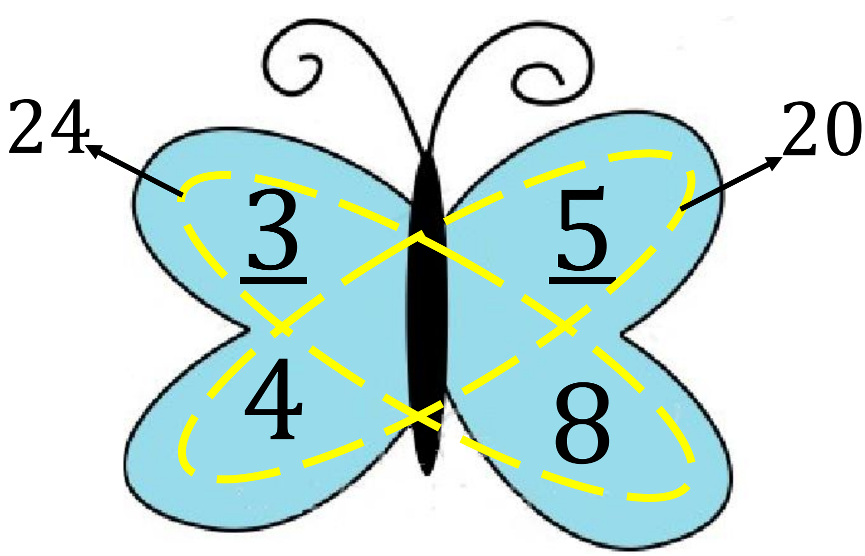
Cross Multiplying (Butterfly Method)

Consider the fractions $\frac{3}{4}$ and $\frac{5}{8}$.

When we draw models for each fraction, we can easily see that $\frac{3}{4}$ is bigger than $\frac{5}{8}$.



We can easily compare two fractions by using the Butterfly Method.



$$24 > 20$$

So, $$\frac{3}{4} > \frac{5}{8}$$

Example: Using the Butterfly Method, fill in each $\square$ with $>, <$ or $=$.

a) $\frac{1}{3} \square \frac{1}{2}$ b) $\frac{3}{8} \square \frac{1}{4}$ c) $\frac{3}{8} \square \frac{2}{5}$

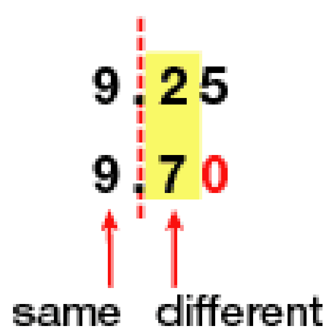
d) $\frac{2}{4} \square \frac{7}{9}$ e) $\frac{3}{5} \square \frac{2}{4}$ f) $\frac{5}{11} \square \frac{4}{7}$

g) $\frac{4}{5} \square \frac{6}{9}$ h) $\frac{4}{5} \square \frac{3}{12}$ i) $\frac{3}{7} \square \frac{2}{5}$

<u>Changing Fractions to Decimals and Comparing Place Value</u>

It is easy to compare decimals using place value.

1. Align the decimal points.
2. Fill in place values with zeros.
3. Compare digits from left to right until they are different.



Since $2 < 7$,

$9.25 < 9.7$.

For example, let's order these decimals from least to greatest!

0.25 1.125 0.537 0.205 1.12

Example: Use place value to compare each pair of numbers by placing <, > or = between them.

a) 0.52 ☐ 0.478

b) 0.143 ☐ 0.14

c) 0.214 ☐ 0.21

d) 1.497 ☐ 0.485

e) 0.77 ☐ 1.077

f) 0.089 ☐ 0.089

g) 1.425 ☐ 1.42

h) 0.112 ☐ 0.12

We can also compare fractions by first changing them to decimals and then using place value.

For example,

a) consider $\frac{6}{10}$ and $\frac{48}{100}$.

Using the methods we learned previously, we know that

$\frac{6}{10} =$ __________ and $\frac{48}{100} =$ __________

b) $\frac{6}{7}$ ☐ $\frac{2}{5}$

Example 1: Compare each pair by converting to decimals and writing a $<$, $>$ or $=$ between them.

a) $\frac{15}{100}$ ☐ $\frac{2}{10}$

b) $\frac{3}{5}$ ☐ $\frac{7}{20}$

c) $\frac{16}{25}$ ☐ 0.7

d) $\frac{87}{100} \square \frac{3}{4}$ e) $\frac{1}{4} \square \frac{573}{1000}$ f) $0.034 \square \frac{1}{25}$

g) $\frac{9}{10} \square 1.083$ h) $\frac{12}{5} \square 2.4$ i) $\frac{3}{20} \square \frac{2}{25}$

Example 2: Change the following fractions to decimal form and order from least to greatest.

a) $\frac{1}{7}, \frac{9}{10}, \frac{7}{9}$

b) $\frac{9}{14}, 2\frac{1}{2}, \frac{12}{7}, 3\frac{6}{8}$

<u>Using Benchmarks to Compare Fractions</u>

We can use the benchmarks $0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}$ and 1 to compare and order fractions.

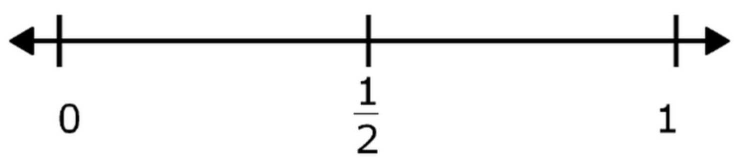
For example, $\frac{2}{15}$ would be closer to 0 since the numerator is much smaller than the denominator, whereas $\frac{7}{15}$ is about $\frac{1}{2}$ since 7 is about half of 15.

Let's consider $\frac{6}{7}, \frac{8}{15}$ and $\frac{7}{30}$.

Using benchmarks, we can order fractions from least to greatest.

$\frac{6}{7} \approx$ ______ $\frac{8}{15} \approx$ ______ $\frac{7}{30} \approx$ ______

We can arrange the fractions from least to greatest on a number line:



<u>Example:</u> Use benchmarks and a number line to order each set of numbers from least to greatest.

a) $\frac{9}{11}, \frac{3}{7}, \frac{1}{10}, 1\frac{1}{4}$

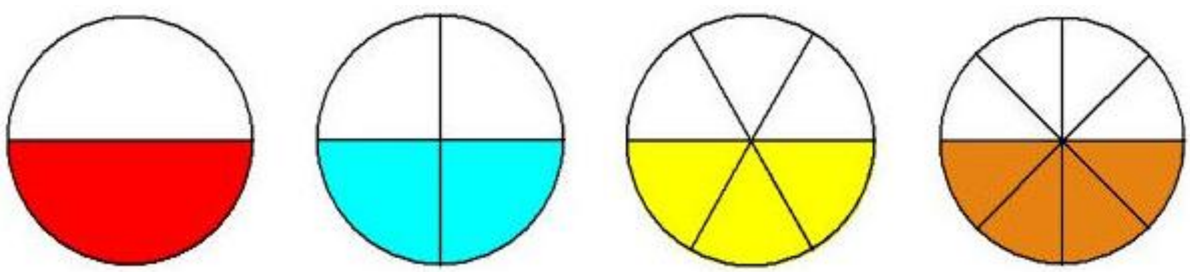
b) $\frac{13}{6}, \frac{3}{5}, 1\frac{7}{8}, 2\frac{1}{3}$

<u>Comparing Common Numerators or Denominators Using Equivalent Fractions</u>

<u>What are Equivalent Fractions?</u>

Equivalent fractions are fractions that have the same value, even though they may look different.

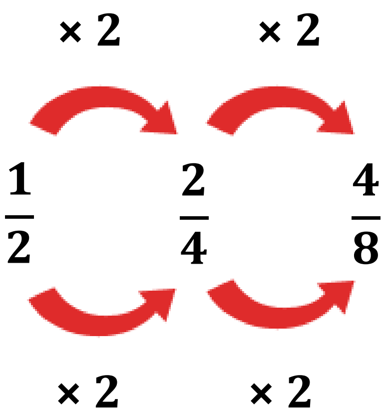
For example,



$\frac{1}{2}$ $\frac{2}{4}$ $\frac{3}{6}$ $\frac{4}{8}$

The above fractions are all equivalent fractions. We can easily see that half of the circle is shaded in each diagram, even though the diagrams look a little different.

We can make equivalent fractions by multiplying (or dividing) the numerator and denominator by the same number.



Change the bottom using multiply or divide ...and the same to the top must be applied!

Example 1: Find the missing value in each fraction below.

a) $\frac{3}{10} = \frac{21}{\quad}$

b) $\frac{5}{30} = \frac{10}{\quad}$

c) $\frac{4}{12} = \frac{20}{\quad}$

d) $\frac{7}{\quad} = \frac{14}{18}$

e) $\frac{4}{\quad} = \frac{44}{77}$

f) $\frac{5}{\quad} = \frac{60}{84}$

g) $\frac{6}{12} = \frac{\quad}{4}$

h) $\frac{15}{\quad} = \frac{30}{40}$

i) $\frac{5}{\quad} = \frac{35}{28}$

We can compare fractions by using equivalent fractions with **common numerators or denominators.**

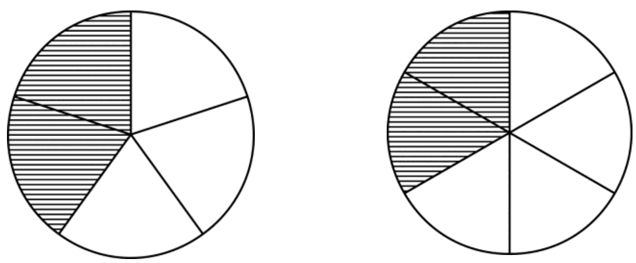
Common Numerators

Let's consider $\frac{2}{5}$ and $\frac{1}{3}$.

We can make both numerators the same simply by multiplying $\frac{1}{3}$ by 2.

So we have $\frac{1\times2}{3\times2}=\frac{2}{6}$

We can easily see that $\frac{2}{5}$ is bigger than $\frac{2}{6}$ or $\frac{1}{3}$**.**



$\frac{2}{5}$ $\frac{2}{6}$

When the numerators are the same, the larger fraction has the smaller denominator.

Example: Compare the fractions using common numerators and writing a $<$, $>$ or $=$ between them.

a) $\frac{3}{5}\,\square\,\frac{1}{2}$ b) $\frac{1}{4}\,\square\,\frac{2}{7}$ c) $\frac{5}{7}\,\square\,\frac{10}{12}$

d) $\frac{4}{5}\,\square\,\frac{2}{3}$ e) $\frac{6}{5}\,\square\,\frac{3}{2}$ f) $\frac{9}{10}\,\square\,\frac{3}{4}$

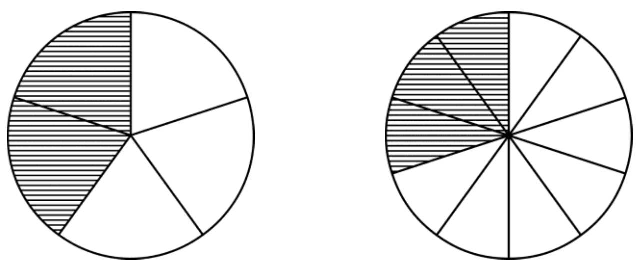
Common Denominators

Let's consider $\frac{2}{5}$ and $\frac{3}{10}$.

We can make both numerators the same simply by multiplying $\frac{2}{5}$ by 2.

So we have $\frac{2\times2}{5\times2}=\frac{4}{10}$

We can easily see that $\frac{2}{5}$ or $\frac{4}{10}$**.** is bigger than $\frac{3}{10}$ **.**



$\frac{4}{10}$ $\frac{3}{10}$

When the denominators are the same, the larger fraction has the larger numerator.

Example 1: Compare the fractions using common numerators and writing a $<$, $>$ or $=$ between them.

a) $\frac{5}{6} \square \frac{11}{12}$ b) $\frac{7}{8} \square \frac{1}{2}$ c) $\frac{2}{3} \square \frac{3}{4}$

d) $\frac{7}{9} \square \frac{5}{6}$ e) $\frac{3}{2} \square \frac{7}{5}$ f) $1\frac{2}{3} \square 1\frac{3}{15}$

Example 2: Order the following fractions from least to greatest using common numerators or denominators:

a) $\frac{2}{3}, \frac{4}{5}$

b) $9\frac{1}{4}, 2\frac{2}{3}, 2\frac{5}{6}$

c) $\frac{5}{6}, \frac{7}{10}, \frac{6}{5}$

Example 3: Find a number between each pair of numbers.

a) $\frac{4}{6}, \frac{5}{6}$

b) $8\frac{2}{3}, 8\frac{1}{3}$

c) $\frac{9}{10}$, 0.92

d) 0.45,0.46

e) $1\frac{3}{5}, \frac{9}{5}$

Section 3.3 – Adding and Subtracting Decimals

Estimation is an important part of mathematics and a very handy tool for everyday life. It should be used to develop a sense of the size of an answer for any calculations involving decimals and to determine whether or not the computations make sense.

There are many ways to estimate an answer. One method is called "Front End Estimation." The name comes from the way that you round; we round to whatever number is in the front.

For example, to estimate

$$9.2 + 3.5 + 12.72$$

we simply use the whole numbers

$$9 + 3 + 12 = 24$$

Once this estimation is complete, the calculation must be performed.

Adding and Subtracting Decimals

To add or subtract decimals, make sure to:

1. Write the numbers down under one another, so that the decimals line up.

2. Add in zeros so that all numbers have the same length.

3. Add or subtract as you normally would, remembering to put the decimal in your final answer.

For example, 1.354 + 1.2

1. Line up the decimals:

```
  1 . 345
+ 1 . 2
```

2. Add in zeros:

```
  1 . 345
+ 1 . 200
```

3. Add normally:

```
  1 . 345
+ 1 . 200
  2 . 545
```

Example 1: Estimate the answer for each of the following then add to determine the exact sum.

a) 2.7 + 9.5

Estimate: ______________

b) 36.2 + 8.9

Estimate: ______________

c) 8.092 + 6.5

Estimate: ______________

d) 6.732 + 8.2 + 13.75 + 6.1564

Estimate: ______________

Example 2: Estimate the answer for each of the following then subtract to determine the exact difference.

a) $9.7 - 3.1$

Estimate: ______________

b) $238.9 - 25.17$

Estimate: ______________

c) $19.005 - 2.9$

Estimate: ______________

d) $20.7 - 6.4 - 2.83 - 0.87$

Estimate: ______________

Practice Problems

1. $4.39 + 18.8 =$

2. $3.68 - 1.74 =$

3. $264.3 + 12.804 =$

4. $116.7 - 32.82 =$

5. $3.75 + 1.08 =$

6. $19.70 + 62.598 =$

7. $21 + 3.814 =$

8. $90 - 25.397 =$

9. $7.52 + 11.77 =$

10. $104.06 - 15.80 =$

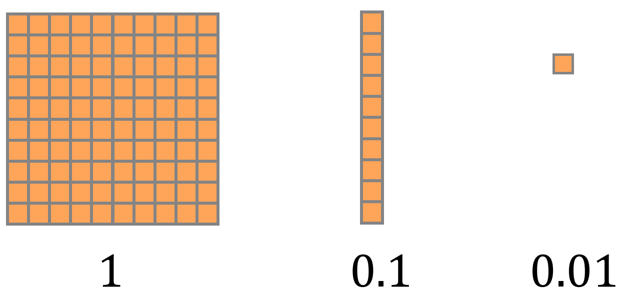
11. Bob has $178.74 in his checking account. He wrote checks for $36.52, $18.92, and $25.93. What is the final balance in Bob's checking account?

12. Simon tallied his bills for building a deck on his house. He paid for the following expenses: labor, $672.25; gravel, $86.77; sand, $39.41; cement, $180.96; and bricks, $204.35. What was the total cost for the deck? If he budgeted $1000, was he over or under budget? By how much?

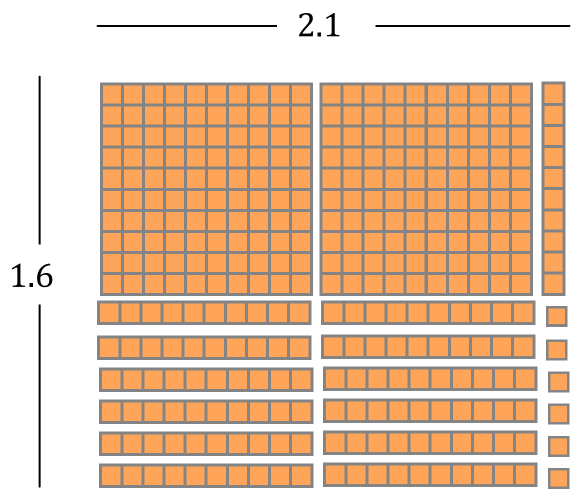
Section 3.4 – Multiplying Decimals

In previous grades, we learned how to multiply whole numbers using base 10 blocks. Now we will extend this method to decimals.

Base-ten blocks:



Let's build a rectangle that has a length of 2.1 and a width of 1.6.

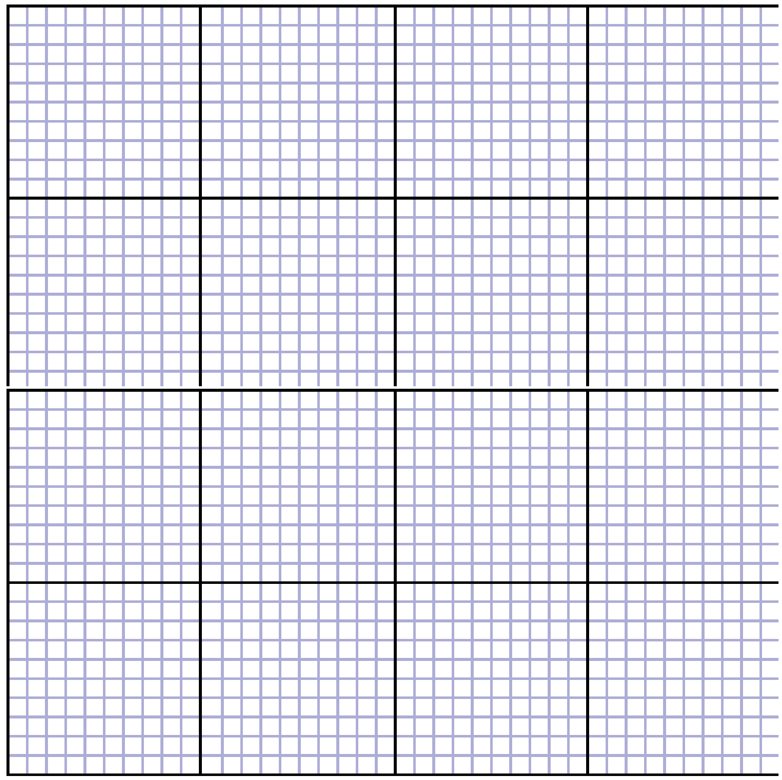


Tally

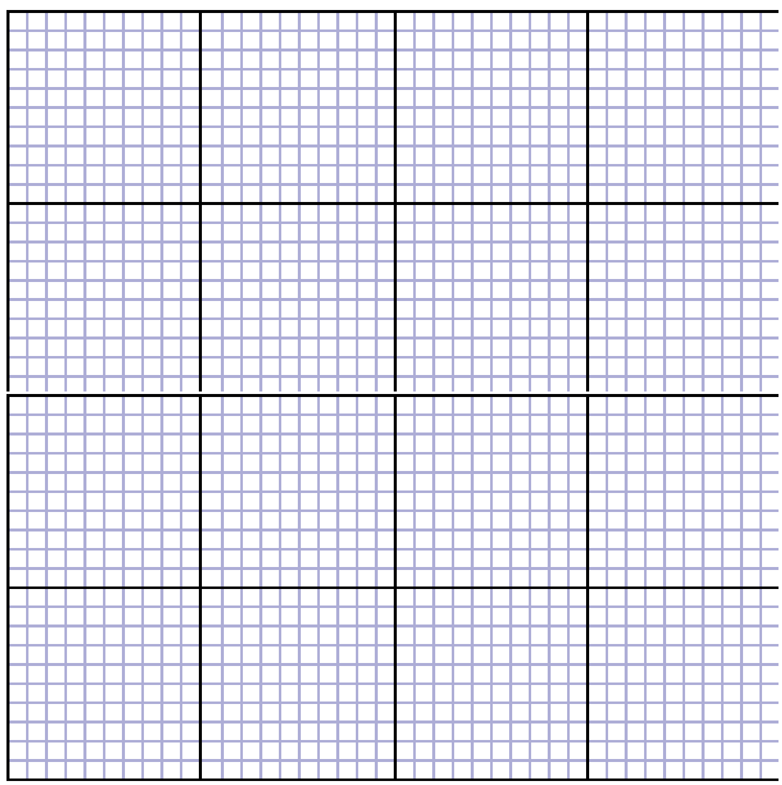
ones	tenths	hundreths

Example: Draw the diagram to show the following. Write the product.

a) 3.1×2.4



b) 1.1×0.3



To multiply decimals without models, we multiply the same way we multiply two whole numbers.

Just follow these steps:

1. Multiply normally, ignoring the decimal points.
2. Put the decimal point in the answer - it will have as many decimal places as the two original numbers combined.

In other words, just count up how many numbers are after the decimal point in both numbers you are multiplying; the answer should have that many numbers after its decimal point.

For example, 0.5×1.3

start with:	**0.5 × 1.3**
multiply without decimal points:	**5 × 13 = 65**
0.5 has **1 decimal place**,	
and 1.3 has **1 decimal place**,	
so the answer has **2 decimal places**:	**0.65**

<u>Example:</u> Multiply the following. Estimate first, and use your estimate to check the reasonableness of your answer.

a) 2.4×1.2

Estimate:_________

b) 6.4×0.12

Estimate:__________

c) 9.8×5.6

Estimate:__________

d) 7.2×0.05

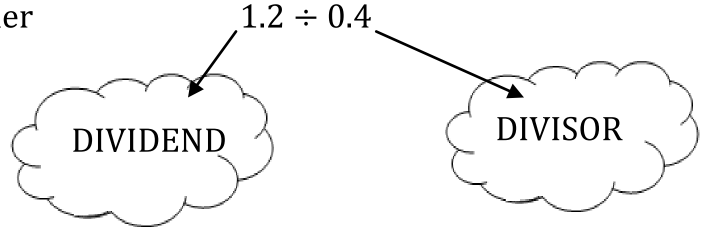
Estimate:__________

Section 3.5 – Dividing Decimals

We can also divide decimal numbers using base 10 blocks.

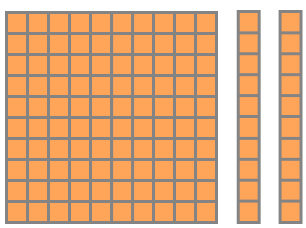
Dividing is the opposite of multiplying.

Let's consider $1.2 \div 0.4$



The divisor(s) are placed on the outside of the rectangle while the dividend is placed on the inside.

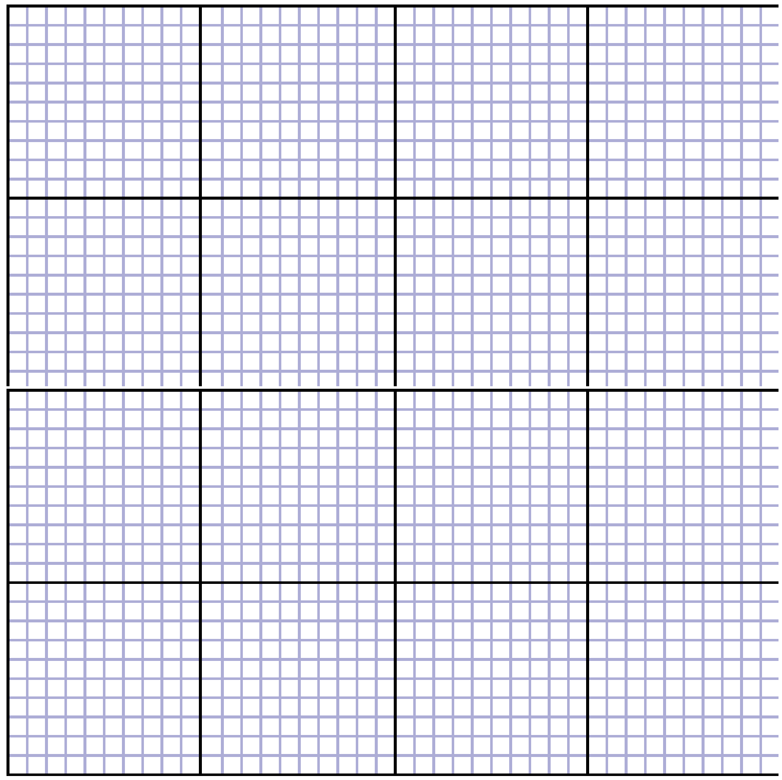
If 2.1 is placed inside the rectangle, how can the blocks be arranged to fit?



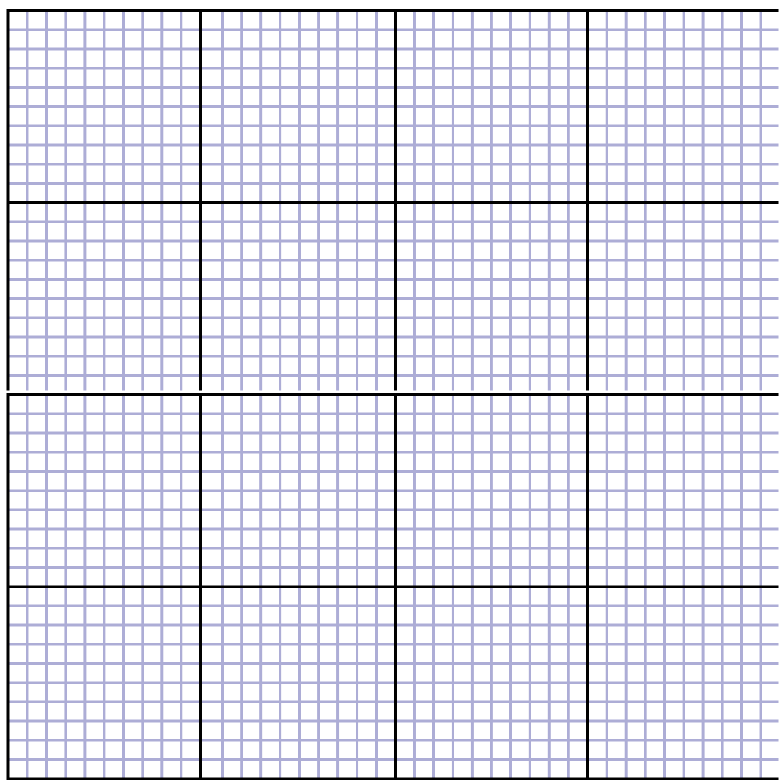
We get

Example: Draw the diagram to show the following. Write the quotient.

a) 2.4 ÷ 1.2



b) 1.6 ÷ 0.8



To divide decimals without models, we have to get rid of the decimal in the divisor. However many places we move the decimal to get rid of it, we have to move the decimal the same number of places in the dividend.

For example,

$3.6 \div 0.6$ means $0.6\overline{)3.6}$ so we move the decimal $0.6\overline{)3.6}$

We get $6\overline{)36}$

<u>Example 1:</u> Divide the following.

a) 5.6 ÷ 0.7

Estimate:_________

b) $3.4 \div 0.4$

Estimate:_________

c) $9.5 \div 6$

Estimate:_________

d) $6.43 \div 0.8$

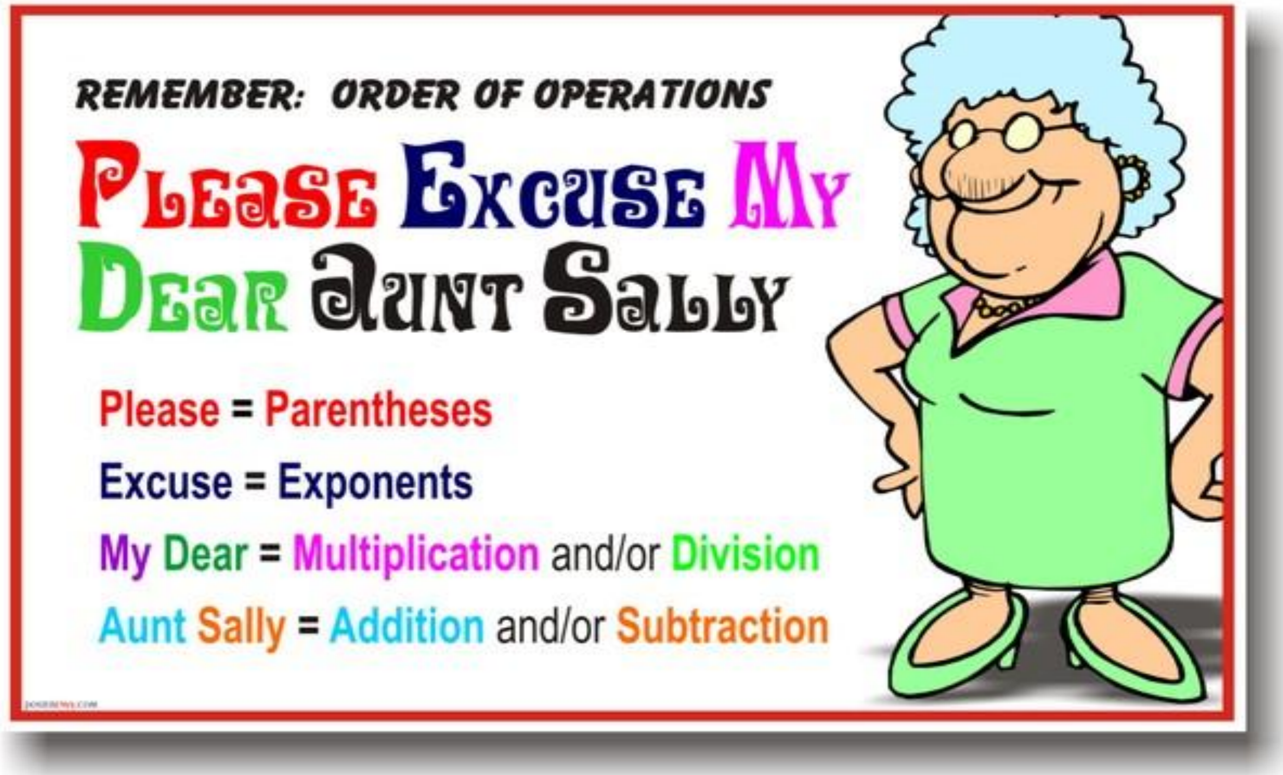
Estimate:_________

<u>Example 2</u>: Use front-end estimation to determine where the decimal should be placed in the following quotients:

A. $39.06 \div 4.2 = 93$

B. $58.5 \div 3.9 = 15$

Section 3.6 – Order of Operations with Decimals



Recall: Multiplication/Division are completed from left to right
Addition/Subtraction are completed from left to right

Examples:

a) $(5.2 + 1.9) \times 0.6$

b) $(3.2 + 6.8) \div (2.5 \times 2)$

c) $2.372 + (9.6 \div 6)$

d) $3.1 + 2.3 \times 4.5$

e) $9.3 \div 3.0 - 1.2$

f) $7.3 \times 2.1 + 5.6 \times 3.6$

g) $(5.8 - 2.1) \times 4.6$

Section 3.7 – Relating Fractions, Decimals and Percents

Percent means per hundred or out of 100.

We know that 0.63 = $\frac{63}{100}$.

Just think, if you received that mark on a test, that would be 63%.
We also know that $\frac{63}{100} = 0.63$ (we move the decimal two places left because we have two zeros).

Example 1: Write the following percents as fractions out of 100 (reduce if possible) and as decimals.

a) 5%

b) 17%

c) 50%

d) $33.\overline{3}\%$

<u>Example 2</u>: Write each as a fraction over 100 and then as a percent.

a) $\frac{2}{10}$

b) $\frac{17}{50}$

c) $\frac{20}{25}$

d) 0.85

e) 0.177

f) 0.35

Example 3: Explain how you would estimate the percentage when a test score is 26 marks out of 55.

Example 4: Determine the percent of a book that is left to read if the class has read 60 out of 150 pages. Explain your thinking.

Section 3.8 – Solving Percent Problems

Percents are used in everyday life – commission, discounts, sales tax and tips at restaurants are just a few examples.

To find a percent of a number we think:

What is 15% of 25?

Remember: $15\% = \frac{15}{100} = 0.15$

In math terms, the word "of" means **multiply**.

This means we now have

$$0.15 \times 25$$

We can multiply this as we normally would.

Example: Find the percent of each number.

a) 8% of 256

What is 8% as a decimal? ___________

b) 98% of 98

What is 98% as a decimal? ____________

c) 102% of 112

What is 102% as a decimal? ____________

Discount is an amount taken off of a normal price.
Sale price is the price that is being paid after the deduction.

For example: Calculate the sale price.

a) A snowboard that costs $99 is on sale for 35% off.

Discount: __________ Sale price: ___________

b) A DVD player that costs $36 is on sale for 68% off.

Discount: __________ Sale price: ___________

Tax is an amount determined by the government on workers' income and business profits or added to the cost of some goods, services, and transactions.

Taxes are calculated the same way as any other percent, using the amount of tax indicated.

__Example:__ Find the total price, including HST (NL: 13%) on each of the following.

a) a pair of skates: $56. (Find 13% of 56)

Tax: ___________ Total: ___________

b) a baseball bat: $31

Tax: ___________ Total: ___________

<u>Example 2:</u> A survey indicated that 42% of the students in a school wanted the cafeteria to add pizza to the menu. If there are 289 students in the school, how many of them want pizza on the menu?

Practice Problems

1. Using NL and Lab sales tax, find the tax for each of the following:

a) a pair of shoes selling for $34.99

b) a hat selling for $20.00

c) a car selling for $12 900

2. Hollie bought a pair of skates for $320.00. How much did the skates cost with taxes?

3. Find the discount on each of the following:

shoes $45.99, on sale 30% off

Playstation $199.99, 20% off

4. Marshall had to buy a scientific calculator for math. He went to Staples and saw a calculator regularly priced at $20.00, with a discount of 30%. He had $20.00 in his pocket. Did he have enough to buy the calculator including taxes?

a) Find the discount.

b) Sale price of calculator

c) Taxes

d) Final price of calculator.

5. Johnny has $450 which he places in a savings account, earning 0.5% per year. Calculate the interest he will earn.

b) If he doesn't withdraw any money, how much will he have in total at the end of the year?

6. Pat is hired as a salesperson for an antique dealer. Pat thinks he can sell $4500 worth of antiques each week. He is offered two methods of payment. 12% commission on sales or 8% of commission on sales plus $100 per week. Which method of payment results in a higher salary, and by how much?